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TIME-SERIES ANALYSIS OF A TWO-HANDED 16TH NOTE DRUMBEAT

Bachelor of Science thesis

Examiner: Prof. Esa Räsänen

ABSTRACT

EEMI FAGERLUND: Time-series analysis of a two-handed 16th note drumbeat
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Fractals are not only beautiful mathematical constructions, but they can be found all around in nature. Temporal fractals characterize a variety of fluctuations in human functions, such as in gait, heart rate and breathing. Fractal behaviour has also been studied in music, and especially in drumming.

In this thesis I first go through the brief history of research regarding fractals in music, and then present some new findings. The fractal nature of musical fluctuations was first discovered in the 1970's, but only in the last ten years or so has its research picked up again. In 2011 fractals in rhythms were observed in laboratory conditions, and afterwards research was conducted on actual recorded music with similar results. A large-scale study in 2018 suggested that fractal fluctuations can be found in music regardless of the style or genre.

In this work we compared the fractal behaviour of a one-handed 16th note drum beat to a two-handed 16th-note drumbeat. It turns out that when both hands are working together to produce a steady rhythm as opposed to just one hand, the fluctuations from a metronome are not that fractal anymore. The long-range correlations previously found in the interbeat intervals are now replaced with clear anticorrelations. We also analysed the amplitude fluctuations of the song in question and found that the dynamic is more long-range correlated in two-handed than in one-handed playing.

Finally we discuss these results from a scientific point of view, but also from the point of view of a drummer. We try to explain these findings using knowledge and experience of the analysed drumbeat, and also from the viewpoint of musical interpretation.

CONTENTS

1. INTRODUCTION AND BACKGROUND

1.1 Fractals

Fractals are shapes and patterns with a self-similar structure [1]. This means that zooming in on a shape does not reveal any new details or features. Instead, a similar (or even identical) structure repeats over and over.

1.1.1 Spatial fractals

Spatial fractals are common in nature. They can be found for example in the way tree branches and broccoli grow, in the rugged shape of a coast line, and in the structure of human lungs. Fractal shapes are often a result of a natural optimization. For example, it is beneficial to have the surface area of lungs as large as possible to most efficiently transfer oxygen into the bloodstream. However, the lungs still need to fit inside the chest. Optimizing this situation results in a fractal structure so that the volume of an adult's lungs is approximately six litres, but their area is roughly that of a tennis court.

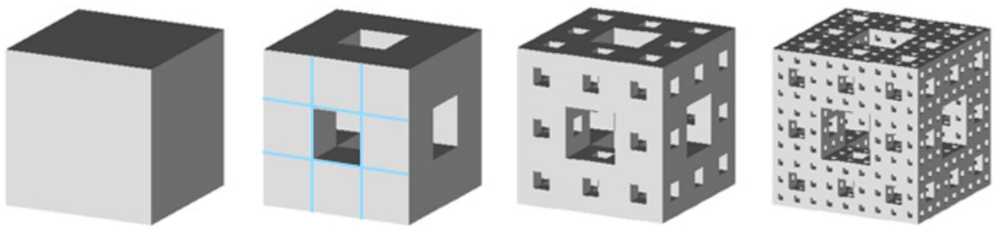

Figure 1.1 *A Menger sponge is created by taking a cube, dividing it into 27 equal smaller cubes, and removing the center cube from each side, as well as the center of the whole cube. Repeating this process for each smaller cube over and over gives a fractal Menger sponge. Image source: fractalfoundation.org*

The scale of natural fractals is of course limited, because they are physical objects made up of atoms. Mathematically defined fractal curves like the Koch curve or the Menger sponge do not have these restrictions and can be scaled infinitely.

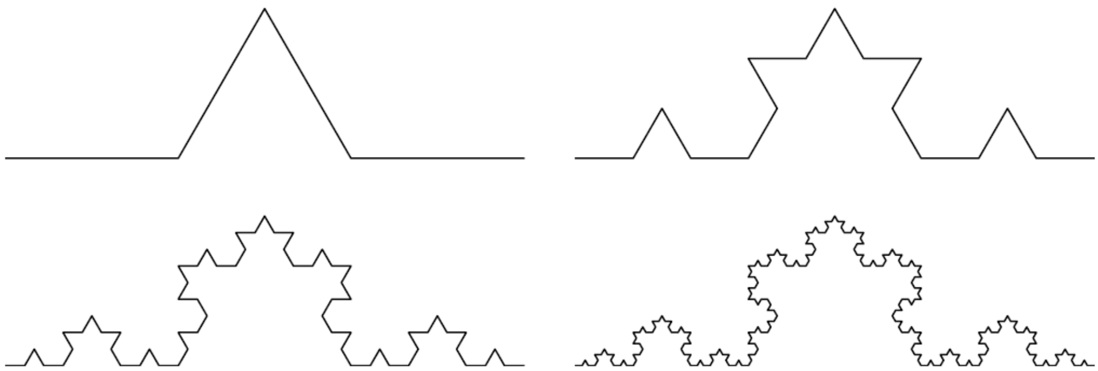

Figure 1.2 *The first four iterations of a Koch curve. It can be made by dividing a line into three equal parts. Then you place an equilateral triangle's base on the middle part and remove said base, then repeat these steps recursively [5].*

For spatial fractals we define the Hausdorff dimension as [13]

$$D = \frac{\ln m}{\ln r}, \tag{1.1}$$

where m is the number of copies in scaling and l is the scaling factor. In a Menger sponge each cube consists of 20 smaller cubes, and each smaller cube has a side length of one third of the bigger one. So we get

$$D = \frac{\ln 20}{\ln 3} \approx 2.73 \tag{1.2}$$

meaning that it is colse to a three-dimensional object.

1.1.2 Temporal fractals

Fractals in time are usually different from fractals in space. Whereas spatial fractals tend have a geometric regularity or symmetry, temporal fractals are more qualitative. An example of a temporal fractal is a heart rate. In Fig. 1.3 we can see that zooming into the graph does not reveal an identically fluctuating pattern, but the graph still retains a similar shape.

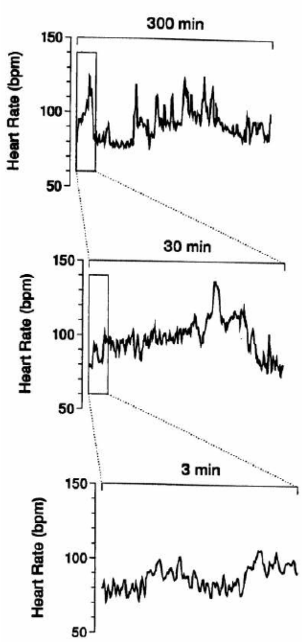

***Figure 1.3** Plot of heart rate as a function of time. Zooming in from 300 minutes all the way to just 3 minutes does not fundamentally change the shape of the plot [10].*

There is randomness in the fluctuations, but the randomness scales according to a power law. Fractally scaling fluctuations are equivalent to $1/f$ scaling in the frequency picture, corresponding to the well-known $1/f$ noise. In fact, a heart rate missing this fractality can be a sign of a heart malfunction, such as a congestive heart failure or cardiac arrhythmia [7]. This thesis focuses on examining temporal fractals found in drumming.

1.2 Fractals in loudness fluctuations and rhythm spectra

Drumming has been a part of human society for thousands of years as a means of communication as well as music. The first study on fractal behaviour in music was carried out by Voss and Clarke in 1975 ̇They examined loudness fluctuations in different radio stations ranging from classical music to rock and news stations. The loudness fluctuation spectra in all cases revealed $1/f$ noise.

Over 30 years later Levitin *et al.* applied the same approach to rhythm spectra [12]. Analysing the spectra of note lengths in various musical pieces showed that throughout Western music - ”from Bach to Joplin” - rhythms obey a $1/f$ power law. Levitin claims in his article:

> *"Much of our enjoyment of music comes from its balance of predictability and surprise. Musical pitch fluctuations follow a $1/f$ power law that precisely achieves this balance."*

1.3 Fractals in rhythmic fluctuations

1.3.1 Laboratory experiments

A laboratory trial on rhythmic fluctuations was conducted in 2011 [4]. Samples for the research were gathered from both amateur and professional musicians playing rhythms ranging from simple to complicated. The subjects produced the rhythms with their hands, feet, and voice. A metronome was used in all cases, tempos being 128 and 180 beats per minute. Regardless of player, rhythm or tempo, long-range correlations (LRCs), i.e., fractal behaviourin the deviations from the metronome, were observed in all cases.

The next step was to examine whether the presence or absence of LRC had any effect on the listener. Subjects were presented with two versions of a song, where one version was humanised using white noise, the other with $1/f$ noise. 79% of the subjects claimed to hear a difference between the two songs. When asked which version they preferred, 64% chose the $1/f$ humanised version.

1.3.2 Music samples

In Ref. [3] the study of LRCs in music was taken a step further. The goal was to try to detect LRCs in a real, professional recording outside of a lobarotary. The examined song was *I Keep Forgettin'* by Michael McDonald. It was released in 1982 and the drums were played by Jeff Porcaro, a legendary drummer known for his work in, e.g., Toto and Steely Dan. The subject of study was Porcaro's right hand hi-hat pattern that can be heard throughout the song. LRCs were found not only in the interbeat intervals, but also in the amplitudes of the hits. An interesting finding was the fact that the correlation in the interbeat intervals disappeared at shorter ranges, at roughly 30 hits or fewer. This scale corresponds to the two-bar phrase that the drummer repeats over and over in the song. However, to prove an actual connection of this transition to the musical structure more research is required.

Another study also found LRCs in recorded music, namely in jazz and rock [2]. A semi-automatic workflow was used here to extract beat onsets from over 100 original recordings with a millisecond precision. This study with a large set of data seems to verify the previous findings of LRCs in rhythmic fluctuations in drumming.

2. MUSICAL SAMPLE FOR THE ANALYSIS

The song chosen for this study was *Cry for Freedom* by the American hard rock band White Lion. It was featured on their 1989 album *Big Game* and the band's drummer at the time was Greg D'Angelo. The drum beat in this song is a 16th note beat that is used throughout the song. Even though some open hi-hat hits and drum fills might disrupt the pattern every now and then, a decent amount of data could be extracted from the song.

D'Angelo's 16th note beat is somewhat similar to the one Porcaro used in I keep Forgettin'. There is however an important, fundamental difference: D'Angelo played the 16th note hi-hat pattern with two hands, alternating between right and left, whereas Porcaro used only his right hand. D'Angelo's choice can be detected from the absence of hi-hat hits on top of the snare. Porcaro has commented on his choice of one-handed versus two-handed version [11]:

> *"I like the single-handed method, because it's a lot smoother feel. For instance in the Michael McDonald record "I Keep Forgettin' ", I tried doing the alternating stroke method of doing 16ths, and it sounded just too stiff and staccato for me."*

Porcaro's quote suggests that there is an audible difference between the two methods of playing the 16th note pattern. This study examines if the fractal nature of the beat changes depending on which method is used.

3. METHODS

3.1 Onset detection

The onsets used in the study were extracted from the original recording in Tuomas Virtanen's research group by Oguzhan Gencoglu at the laboratory of Signal Processing in TUT. The recording had a sampling rate of 44.1 kHz and a bit depth of 16 bits per sample. The problem in using an actual recording, however, is that all the instruments are mixed together, so we need a method to isolate specific components. In this case we were interested in the hi-hat and snare hits, e.g. what the drummer does with his hands.

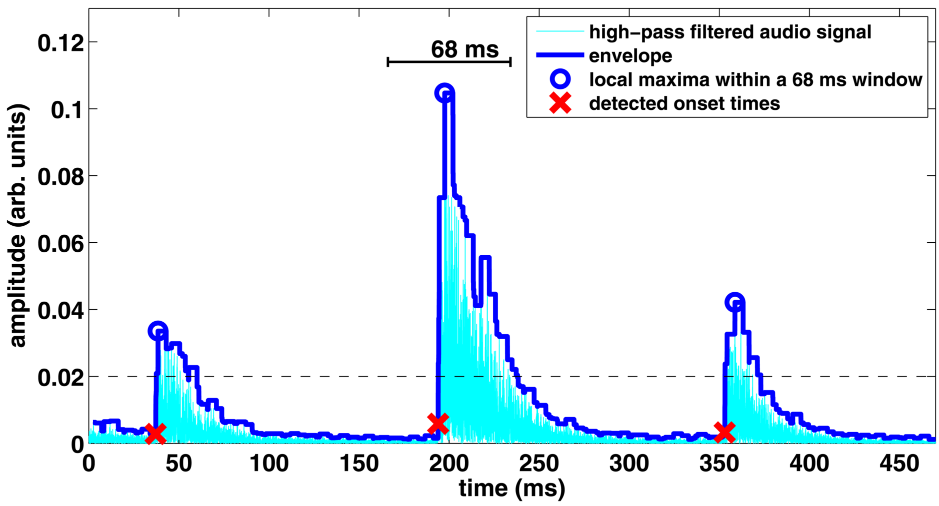

Figure 3.1 *Onset detection from the filtered signal. Local maxima in the envelope (dark blue) are detected, and 3000 sample windows around each peak are searched, determining the onset time at 10% of the maximum (red cross). The sample is not from the song that was used in this study, but the onset detection method is the same [3].*

A hi-hat and a snare typically have a high pitch, so they are already relatively isolated at higher frequencies. To make onset detection easier the audio sample was first filtered with a 100th-order finite impulse response (FIR) filter with a cutoff frequency of 8 kHz. Next an envelope was calculated for the filtered signal by taking the absolute value of the signal and finding the maximum within a 200 sample (4.5

ms) window. The onsets can now be seen as distinct spikes in the envelope (Fig. 3.1). To determine the exact onset times a 3000 sample (68 ms) window around each envelope maximum was examined.

An onset was defined to occur when the envelope reaches 10% of its maximum amplitude. If interference from other instruments raised the general level of the envelope above the 10% threshold, the threshold was doubled until it was above the envelope again. After the onsets had been extracted, they were manually inspected to make sure they correspond to the perceived hits.

3.2 Detrended fluctuation analysis

Detrended fluctuation analysis (DFA) is a method to determine a scaling exponent α for a time series. First we take the series and subtract its mean from every element

$$\hat{x}_i = x_i - \langle x_i \rangle, \tag{3.1}$$

so that we get a new series with a zero mean. Next, we calculate the cumulative sum of this new series:

$$Y(j) = \sum_{i=1}^{j} \hat{x}_i, j = 0, 1, 2, \ldots, N. \tag{3.2}$$

This cumulative sum is then divided into N_s non-overlapping windows of size s. A degree m polynomial is fit to each window using the leasts square estimate. Next we calculate the residuals

$$\tilde{Y}(j) = Y(j) - y^m_{v,s}(j), \tag{3.3}$$

and finally the root-mean-square fluctuations for every window size s:

$$F^2_{\mathrm{DFA}m}(v, s) = \frac{1}{s} \sum_{j=1}^{s} \tilde{Y}^2(j). \tag{3.4}$$

These quantities are further averaged, and taking a square root leads to

$$F_2(s) = \sqrt{\frac{1}{N_s} \sum_{v=1}^{N_s} F^2_{DFAm}(v, s)}. \tag{3.5}$$

This process is repeated for many different window sizes. Plotting the average fluctuations as a function of window size in a log-log scale should then give a straight line. From the slope of this line we can get the scaling exponent α, so that

$$F_2(s) \propto s^{\alpha}. \tag{3.6}$$

Values of α relate to correlations in the following way:

$0.5 < \alpha \leq 1.5$	*long-range correlations*
$\alpha = 1$	*“fractal” (1/f) noise*
$\alpha = 0.5$	*white noise*
$-0.5 < \alpha < 0.5$	*anticorrelations*

4. RESULTS

4.1 Interbeat intervals

Knowing that LRCs have already been found in interbeat intervals of *one-handed* 16th-note hi-hat patterns [3] one could make the assumption that LRCs in intervals could be observed in the *two-handed* case too. In Fig. 4.1 we show the DFA results for the interbeat interval fluctuationd $F(s)$ as a function of the scale s.

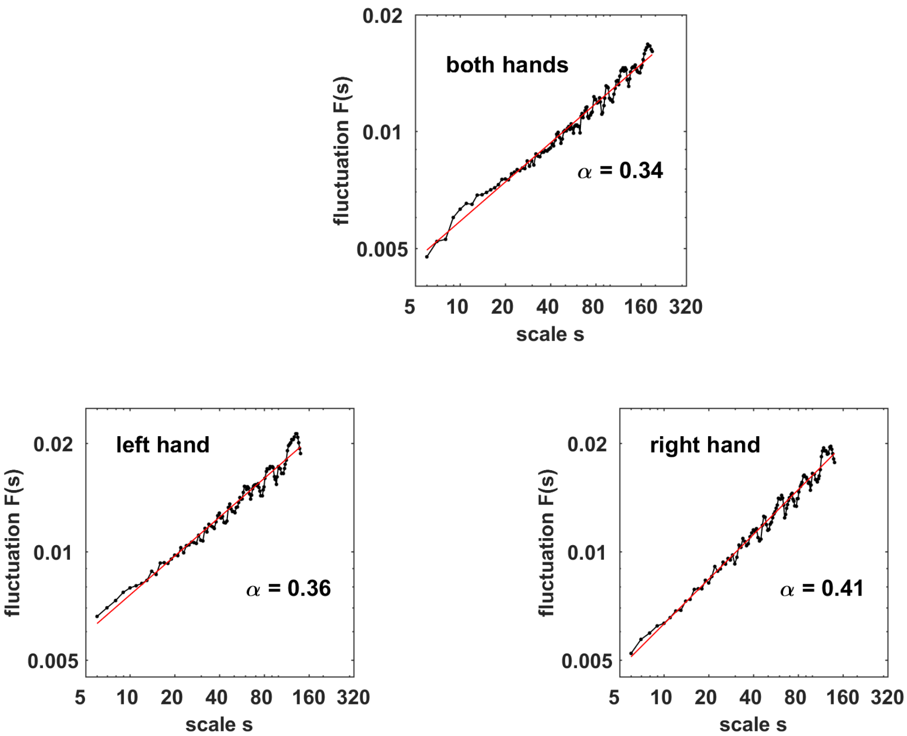

Figure 4.1 *Results of the detrended fluctuation analysis with the scaling exponents α for interbeat interval fluctuations. Contrary to previous studies, no long-range correlations were detected.*

The results show that the scaling exponents are 0.36 for left hand, 0.41 for right hand and 0.34 for both. Considering that previous studies seem to be unanimous about LRCs in music these results are surprising. In essence, LRCs are not observed in the two-handed pattern, and even when examining the hands separately, the fluctuations in interbeat intervals seem to be slightly anticorrelated accross the scale. Does this suggest that even though one-handed patterns have been observed to be fractal, alternating between two hands destroys that fractal behaviour? Additional research is required to make any definitive conclusions.

Poincaré plots of intervals (Fig. 4.2) show that in the case of both hands and only the left hand the intervals are clearly anticorrelated. Since a drummer constantly tries to maintain a steady beat it is only natural that longer intervals are usually followed by shorter intervals and vice versa. The right-hand Poincaré plot on the other hand is rather peculiar. The correlation coefficient is extremely close to zero indicating no particular correlation. This is especially interesting because the right hand is the dominant, "leading" hand and as established earlier much more active thought goes into the hits played with the right hand in a two-handed 16th note beat. So intuitively the right hand should be strongly subject to small stabilising actions resulting in significant anticorrelation.

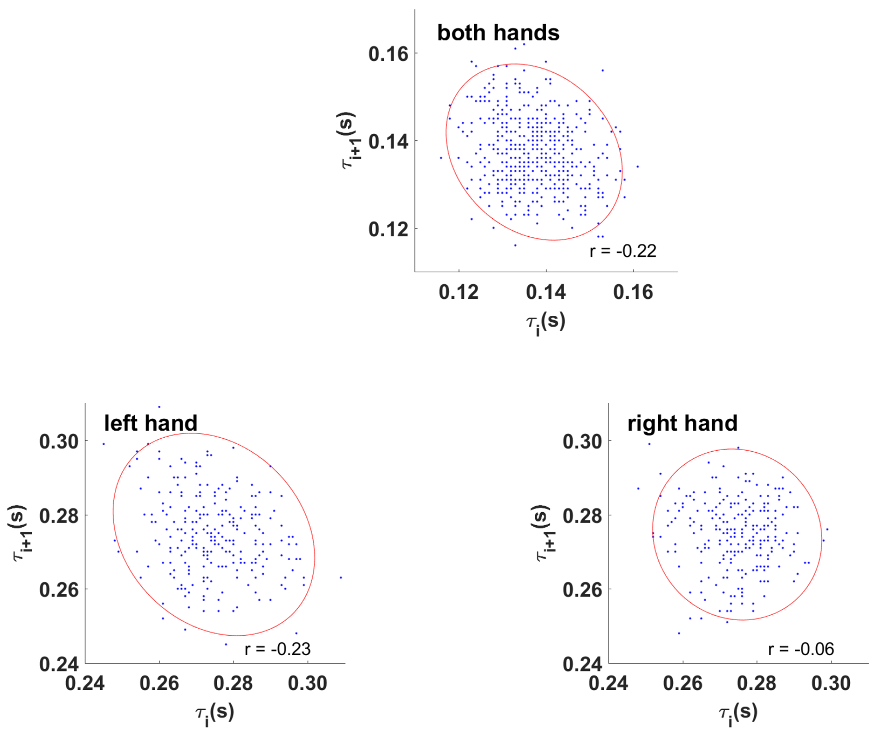

Figure 4.2 *Poincaré plots for interbeat intervals. Each figure shows anticorrelated behaviour. This is expected, since staying in rhythm is a self-correcting process.*

Fig. 4.3 shows the distributions of the interbeat intervals. They seem to follow the normal (Gaussian) distributions fairly well. This makes sense, since playing in a particular tempo means that the drummer tries to keep a steady interval between the hits. Fluctuations from that tempo are inevitable and smaller fluctuations occur more often than larger ones.

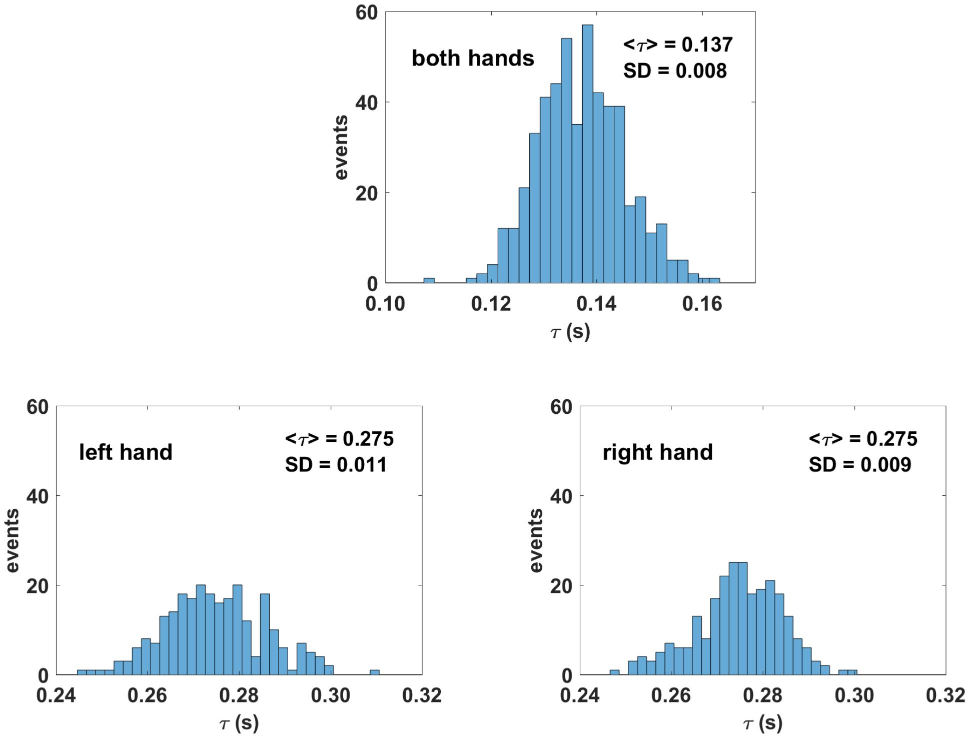

***Figure** 4.3 Distributions of interbeat intervals. Since a drummer will try their best to keep a steady tempo, it makes sense that deviations from that tempo have an approximate Gaussian distribution.*

Next we consider the tempo drift, i.e., the cumulative sum of the interbeat interval fluctuations with respect to the mean. Te results are shown in Fig. 4.4 The first thing to notice is the fact that the drummer starts falling behind. At most this difference from the average is almost half a second. This is a clear indicator that a metronome was not used while recording the song.

The integrated fluctuations are also dependent on the structure of the song. Fig. 4.4 has been divided into sections with dashed lines and letters A for the verse, B for the chorus and C for the interlude.

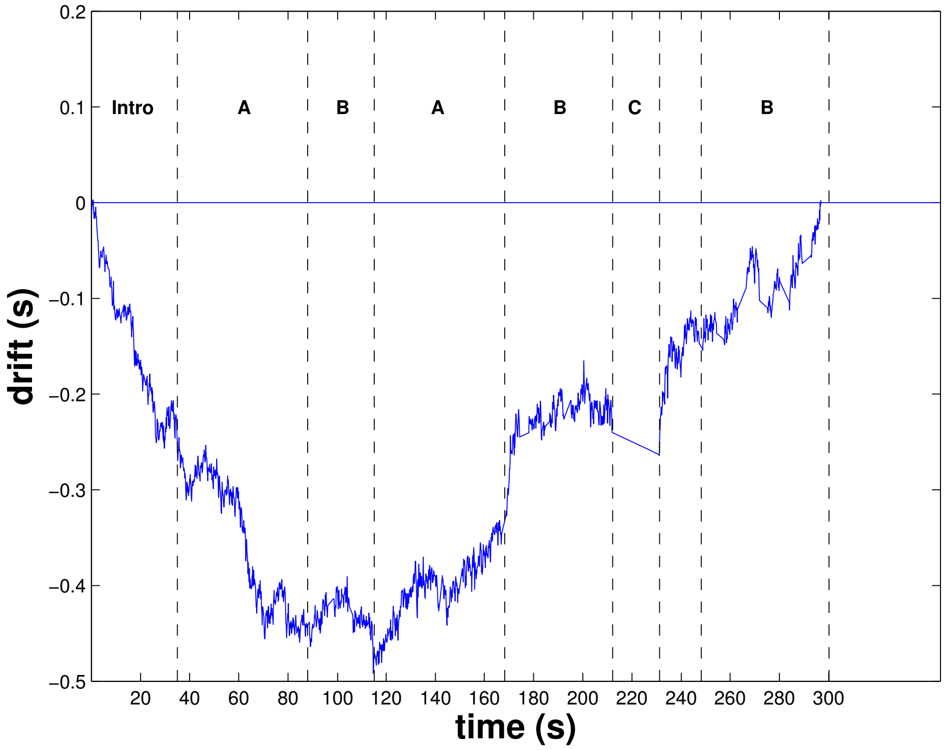

Figure 4.4 *Fluctuation of tempo during the song. Adding the structure of the song to the figure shows how the tempo changes from one section to another.*

We can see that the falling behind goes on until the second verse and from there on the drummer starts playing faster. This can be explained by the artistic aspects of music. Musicians are human and their personal feelings and interpretation affects and should affect their playing. The beginning of a song might be more ”laid-back” so they play more calmly but accelerate when the chorus starts and the general energy of the song goes up. This can be seen in *Cry For Freedom* in the way that the deceleration momentarily stops right at the beginning of the first chorus. Then the tempo takes a noticeable leap from the second verse to the chorus.

4.2 Amplitudes

Let us examine the amplitude variations within a bar of the song. In Fig. 4.5 our amplitude data has been divided into 32 bars of 16 beats each and color coded so that blue, green, and red asterisks represent the right hand on the hi-hat, the left hand on the hi-hat and the right hand on the snare, respectively. Also an average amplitude for each beat is illustrated as a black line.

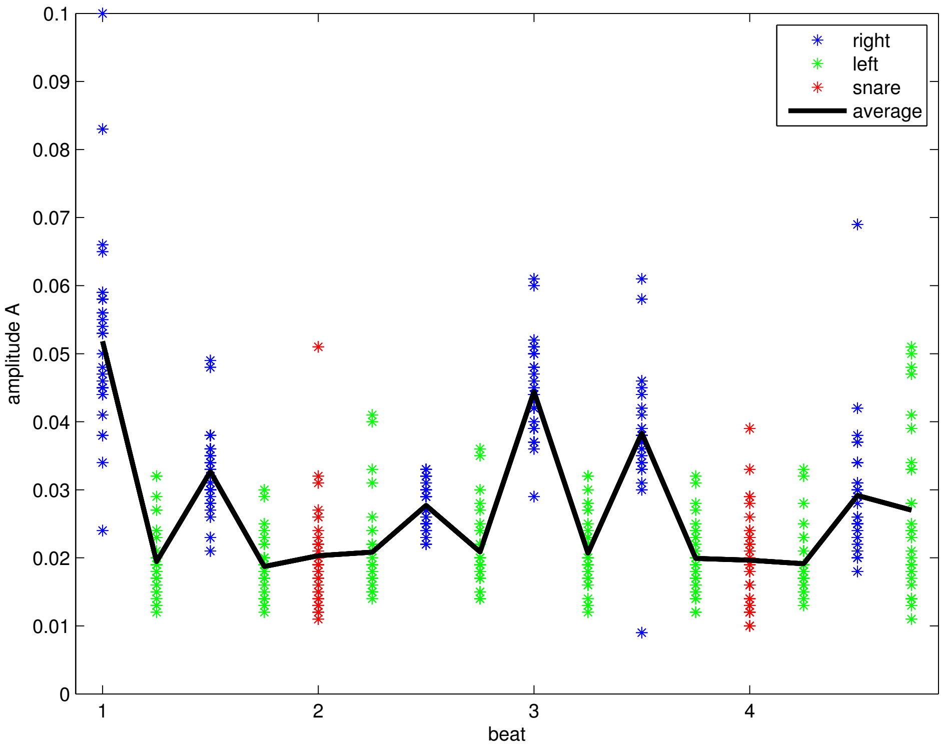

Figure 4.5 *Onset amplitudes arranged by their place in a bar. Hits with the right hand are clearly more accented and they also have a larger dynamic range. Snare hits appear softer than they should be, most likely due to the high-pass filtered data.*

The loudest hits occur at the beginning of each half of the bar, i.e., on the first and 9th hits, both played with the right hand. Everything else is played noticeably softer, and furthermore, the right hand seems to be overall louder than left, with the exception of the snare hits which are as quiet as the left-hand hi-hats. This is most likely due to the fact that the onset data has been extracted from frequencies which best bring out the hi-hat. Since this is not the best frequency range to hear the snare drum the hits do not appear as loud as they should.

In Fig. 4.6 we show the amplitude distributions for each hand. These distributions support the data in Fig. 4.5: The distribution for the left hand is quite narrow, whereas for the right hand it is much more evenly spread with its average clearly above the left-hand average.

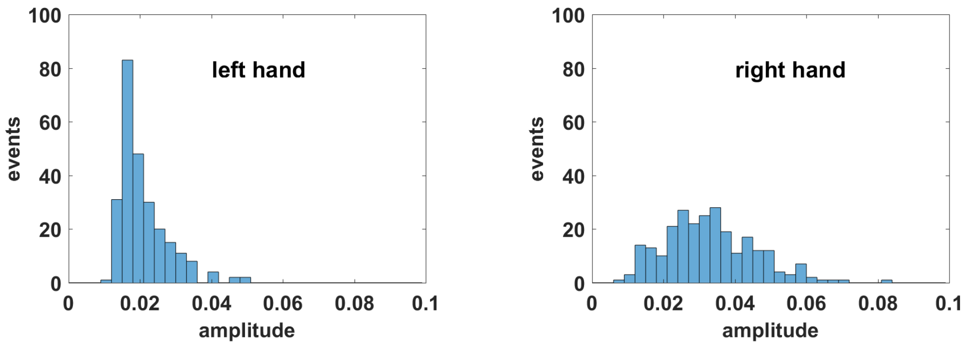

Figure 4.6 *Amplitude distributions for the hands separately. The right-hand amplitudes have an almost Gaussian shape, whereas the left hand has a much more narrow spread with most of the hits at low amplitudes.*

Moving on to the Poincaré plots of the amplitudes we can see that they are consistent with what we have seen so far. In Fig. 4.7 we each amplitude to the following one for both hands together as well as separately. For every plot an ellipse has been fit to contain 95% of the data points, and also a correlation coefficient has been calculated.

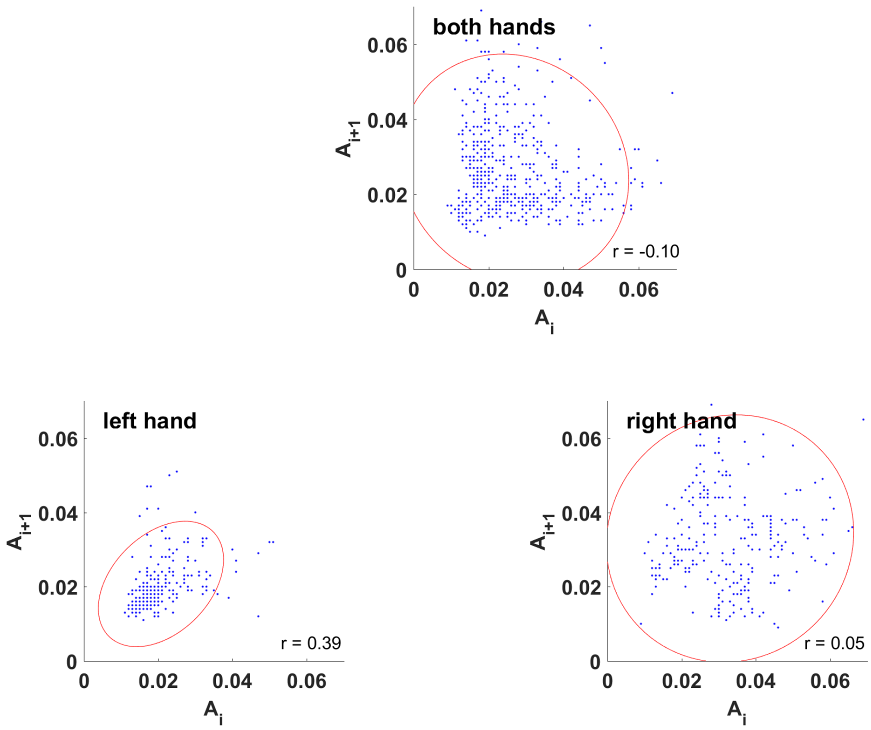

Figure 4.7 *Poincaré plots for the amplitudes. Amplitudes of the left hand have some clear correlations, whereas the right hand has practically no correlations. Both hands together show slight anticorrelations.*

When examining the amplitudes of both hands we can see that the amplitudes are slightly anticorrelated. This can be easily seen also in Fig.1 as a louder hit with the right hand is always followed by a softer hit with the left hand. It can be assumed that without the error in the snare amplitudes the anticorrelation would be even greater.

By comparing the separate hands with each other we can notice that both are slightly correlated, although very little in the case of the right hand. Once again, the right hand is affected by the error in the snare amplitudes, and in reality they might actually be anticorrelated. The left hand has a clear correlation. This indicates that all the accents and other "artistic interpretation" within the 16th-note beat is done with the right hand and the left hand just fills the remaining gaps, so that its amplitude slowly drifts back and forth.

The DFA results in Fig. 4.8 show that the amplitude fluctuations in the both hands seem to be close to white noise, but slightly anticorrelated with fractal exponents 0.48 and 0.49 for left and right hand, respectively.

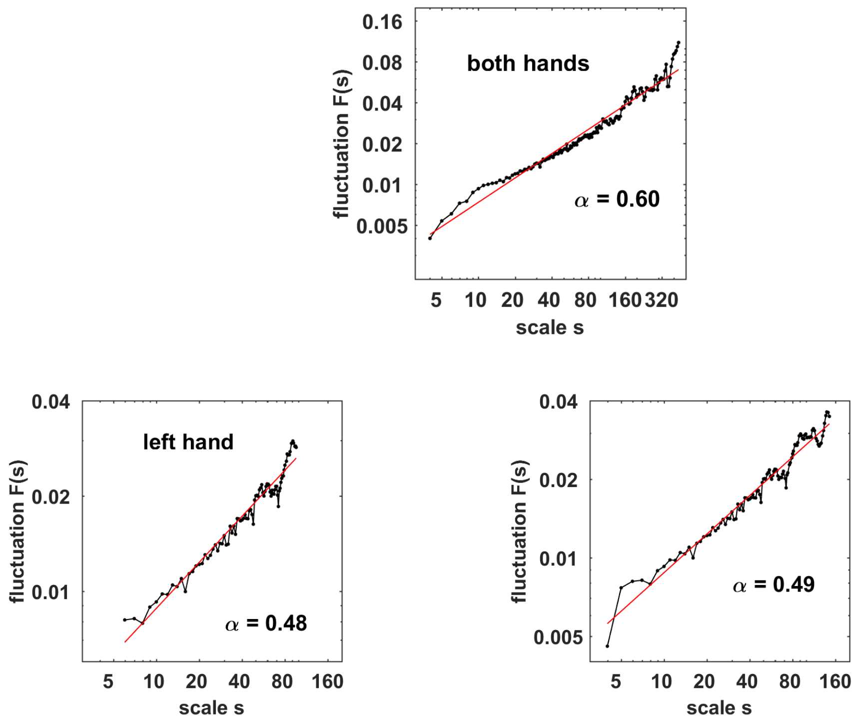

Figure 4.8 *Results of the detrended fluctuation analysis with the scaling exponents α for amplitude fluctuations. Separate hands show slight anticorrelations, but together they have some long-range correlations.*

However, when analysing the both hands together the fractal exponent is 0.6, which is in the regime of LRCs. One could argue that at least three different values of α at different scales could be fitted to the plot of two-handed amplitudes. However, a more detailed analysis of the behaviour of α at different scales is beyond the scope of this work.

5. CONCLUSIONS

The focus of this work was on the time-series analysis of the drum track in the rock song *Cry for Freedom* by White Lion. The results suggest that the previously observed fractal scaling in rhytmic fluctuations disappears when a rhythm is played with alternating hands, as opposed to one hand. In addition, the fractal scaling disappears also from the fluctuations of a one-handed pattern when it was a part of a more complicated rhythm. Some fractal scaling was found in the amplitude fluctuations of the two-handed pattern, but again none could be found when examining the drummer's hands separately.

The Poincaré plots for interbeat intervals show anticorrelated behaviour between consecutive hits. This was to be expected, since the drummer tries to keep a steady tempo and instinctively compensates for interbeat intervals that are shorter or longer than the mean.

The $1/f$ noise in rhythmic fluctuations of human playing has been observed in multiple studies, in many different circumstances. In this respect, our different results for the drumming pattern played with two hands are interesting. So far, the reasons for the disappearing fractal scaling are largely unknown, and little (if any) studies have been conducted on alternating hand patterns. In that regard these results should motivate further research. For example, a laboratory study on two-handed drum patterns would be useful to verify the findings in this thesis. Also, experimenting on different drummers, different patterns and previously unstudied tempos, e.g., extreme drumming with 16th notes at around 200 beats per minute could give valuable insight into the fractal scaling in music in a broader context. Drummers also use their feet while playing, so it would be fascinating to study the effect of additional limbs.

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